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CLVI.

(Vol. VII. March, 1878.)

ON A NEW METHOD OF DETECTING OVERSTRAIN IN IRON AND OTHER METALS, AND ON ITS APPLICATION IN THE INVESTIGATION OF THE CAUSES OF ACCIDENTS TO BRIDGES AND OTHER CONSTRUCTIONS.

A Paper by ROBERT H. THURSTON, C. E., Member of the Society.

READ MARCH 20TH, 1878.

It has been shown by the writer* and by other investigators that, when a metal is subjected to stress exceeding that required to strain it beyond its original apparent, or "primitive," elastic limit, this primitive elastic limit becomes elevated, and that strain-diagrams obtained autographically, or by carefully plotting the results of well conducted tests of such metal, are "the *loci* of the successive limits of elasticity of the metal at the successive positions of set."†

It has been shown by the writer also that, at the successive positions of set, strain being intermitted, a new elastic limit is, on renewing the application of the distorting force, found to exist at a point which

* See Trans. Am. Soc. C. E., 1874, *et seq.*, Journal Franklin Institute, 1874; Van Nostrand's Eclectic Engineering Magazine, 1874, etc., etc.

† On the strength, etc., of Materials of Construction. 1874, Sec. 20.

approximately measures the magnitude of the load at the moment of intermission.*

It has been still further shown by the writer, and by Commander Beardslee, U. S. N., by direct experiment in the Mechanical Laboratory of the Stevens Institute of Technology, and at the Washington Navy Yard, that the normal elastic limit, as exhibited on strain-diagrams of tests conducted without intermission of stress, is exalted or depressed when intermission of distortion occurs, according as the metal belongs to the iron or to the tin class.† This elevation of the normal elastic limit by intermitting strain is, as has been shown, variable in amount with different materials of the iron class and the rate at which this exaltation progresses is also variable. With the same material and under the same conditions of manufacture and of subsequent treatment, the rate of exaltation is quite definite and may be expressed by a very simple formula. The writer has experimented with bridge material, and Commander Beardslee has examined metal specially adapted for use in chain cables, for which latter purpose an iron is required, as in bridge building, to be tough as well as strong and uniform in structure and composition. The experiments of the latter investigator have extended to a wider range than have those of the writer, and the effect of the intermission of strains considerably exceeding the primitive elastic limit has been determined by him for periods of from one minute to one year.‡ From a study of the results of such researches and from a comparison with the latter investigation, which was found to be confirmatory of the deduction, the writer has found that, with such iron as is here described, the process of exaltation of the normal elastic limit due to any given degree of strain usually nearly reaches a maximum in the course of a few days of rest after strain, its progress being rapid at first and the rate of increase quickly diminishing with time. For good bridge irons, the amount of the excess of the exalted limit, as shown by subsequent test, above the stress at which the load had been previously removed may be expressed approximately by the formula :

$$E^1 = 5 \text{ Log. } T + 1.50 \text{ per cent.};$$

in which the time, T, is given in hours of rest after removal of the tensile stress which produced the noted stretch.

* On the Mechanical Treatment of Metals ; Metallurgical Review, 1877 ; Engineering and Mining Journal, 1877.

† Trans. Am. Soc. C. E., 1877.

‡ The result on this investigation is completed and will be presented to the President of the United States by the United States Board appointed to test iron, steel, etc.

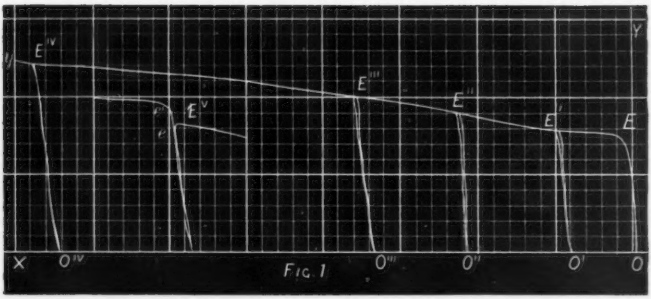

FIG. 1

Thus, in the figure, which is a facsimile of a part of a strain-diagram produced by such an iron, during a test in which the intermission of stress was of too brief duration to cause an observable exaltation of the normal elastic limit in a diagram drawn on so small a scale, the point E is the primitive elastic limit of the material, and E^{I} E^{II} E^{III} E^{IV}, are the normal elastic limits corresponding to sets under loads which have strained it beyond that primitive elastic limit. In the example here illustrated, the primitive limit is found at about 20,000 pounds per square inch, or 1,400 kilograms per square centimeter, and the other points are those corresponding to loads of, respectively, 21,000, 22,500, 25,000 and 30,000 pounds on the inch, or to 1 470, 1 575, 1 750 and 2 100 kilograms on the square centimeter. The corresponding extensions, as shown on the diagram, are 1.25, 2.53, 4.50 and 6.78 per cent.

Had the stress been intermitted at either of these points any considerable period of time, there would have been observed a rise in the diagram as above stated like that shown in Fig. 1, at E^{V} the normal elastic limit e, being, on subsequent test, found altered and a new limit, e', observed. The extent of this elevation of the limit would be the greater as the time of rest was greater, as already seen.

Thus, it is seen that a metal, once overstrained, carries, permanently, unmistakable evidence of the fact* and can be made to reveal the amount of such overstrain at any later time with a fair degree of accuracy. This evidence cannot be entirely destroyed, even by a moderate degree of annealing. Often, only annealing from a high heat, or reheating and reworking, can remove it absolutely. Thus, too, a structure, broken

* The writer has found by subsequent tests, that transverse strain produces the same effect upon the elastic limit for tension.

down by causes producing overstrain in its tension members, or in its transversely loaded beams (and, probably, in compression members—although the writer is not yet fully assured of the latter), retains in every piece a register of the maximum load to which that piece has ever been subjected; and the strain-sheet of the structure, as strained at the instant of breaking down, can be thus laid down with a fair degree of certainty.

Here, then, when the work above detailed shall have been properly complemented with experimental determinations of the behavior of all the materials of general use in construction, may be found a means of tracing the overstrains which have resulted in the destruction or the injury of any iron or steel structure, and of ascertaining the cause and the method of its failure, in cases frequently happening in which they are indeterminable by any of the usual methods of investigation.

The fact of the normal variation of the elastic limit, as change of form progresses under gradually increased load, has been well established by the experiments of Hodgkinson, Clark, Mallett, and other English investigators; by Tresca, particularly, in France; by Werder and Bauschinger in Germany, and by Beardslee, the writer and others in the United States.

The exaltation of the series of normal limits so produced, still further, as shown by the writer and as seen in Fig. 1, by the intermission of strain, as at E^{v}, is also a matter of no uncertainty as to its character, although much more study is needed to determine the modifying effects of time of intermission on metals of the two great classes and of differing composition. The method above outlined of determining the extent of previous overstrain may therefore be expected to have many useful applications.

In illustration of an application of the facts thus reviewed to the determination of the causes and the method of the injury or the destruction of a structure, assume the existence of a set of conditions which is familiar to, probably, every engineer in the country who has seen much of the Howe truss, and of some other forms of bridges, as frequently built before the present generation of professional bridge builders effected a revolution in that department of engineering construction.

Suppose one of these bridges to have been built with a span of 150 feet and to have been given such proportions that, with a weight of 1,200 pounds per running foot and a load of one ton per running foot, the maximum stress on end-rods, or other members most strained, is as high

as 20 000 pounds per square inch of section of metal. Suppose this bridge to have its tension members composed of a fair, but unrefined, iron, having an elastic limit at about 17 000 pounds per inch, and a tenacity of 45 000 to 48 000 pounds, and with an extensibility of about 20 per cent.

Suppose this structure to break down under a load exceeding that usually sustained in ordinary work, and the cause of the disaster to be "involved in mystery."

Suppose portions of the several tension members to be subsequently removed, and, a few days after the accident, to be carefully tested, with the following results :

	Elastic Limit.	Tenacity.
Sample No. 1	16 500	46 000
" " 2	18 000	48 000
" " 3	30 000	48 000
" " 4	22 500	50 000
" " 5	25 000	52 000
" " 6	27 500	52 000
" " 7	28 000	52 000
" " 8	30 000	52 000
" " 9	32 000	53 000
" " 10	34 000	53 000

And that the extensibility is found to be as little as from ten to fifteen per cent.

Suppose it to be found that the tension members were straight bolts without upset ends, the threads being cut, as was once common, in such a manner that the section at the bottom of the thread is one-third less than the sectional area of the body of the bar. Suppose, finally, that the location of the tested pieces in the structure being noted, it is found that the stronger metal, having also the highest elastic limit, came from the neighborhood of the point at which the bridge gave way, and that the weakest metal and that exhibiting the lowest elastic limit came usually from points more or less remote from the break. It is not likely that in all cases the increase in the altitude of the elastic limit and the increase noted in the ultimate strength of the samples would exhibit a regular order coincident with the order of the rods as to position in the structure ; since the magnitude and the arrangement of the bars would, to a certain extent, determine the relative amounts of strain thrown upon them by overloading any one part of the truss. For present purposes we may assume the order of arrangement to be thus coincident.

On examination of the figures as above given, the engineer would conclude : First, that the original apparent elastic limit of the iron used

in this case must have been not far from 17 000 pounds per square inch and that its tenacity was between 46 000 and 48 000 pounds; secondly, that this primitive elastic limit had been elevated, by subsequent loads exceeding that amount, to the higher figures given by the bars numbered from 3 to 10 inclusive; thirdly, that the ultimate strength of the material had been, in some examples given above, increased by similarly intermitted strain.

It would be concluded that the ordinary loads, such as had been carried previously to the entrance upon the bridge of that which caused its destruction, never exceeded, in their straining action, 16 500 pounds per square inch of section of tension rod at the part of the truss from which No. 1 had been taken, and that the other rods tested had carried, probably, at the time of the accident, loads approximately equal to those required to strain them to the extent measured by their elastic limits at the time of testing them.

It would be concluded that the rod from which No. 10 was cut was either that most strained by the load, and therefore nearest the point of fracture of the truss, or that it was very near that point, and it would be made the basis of comparison in further studying the case.

As this elastic limit approaches most nearly the breaking strength of the metal, we may apply the formula for the elevation of the elastic limit with time after intermitted strain which has been above given as derived from tests of a metal of very similar quality. Taking the time of intermission as one week, the extent of the increase has a probable value not far from $E = 5 \log. 168 + 1.5 =$ nearly 12½ per cent. The magnitude of the stress upon this piece at the time of the accident was therefore 34 000 less one-ninth of that value, or about 30 000 pounds per square inch of cross-section of the bar. This corresponds to about 45 000 pounds per square inch at the bottom of the thread, and is within five per cent. of the primitive breaking strength of the iron. The bar, if broken at the screwed portion, has therefore yielded either under a dead load which was at least equal to its maximum resistance, or under a smaller load acting so suddenly as to have the effect of a real "live load." Or the slight difference here noted may be due to a flaw at the point of fracture. However that may be, it is almost certain that the body of the rod has sustained a stress of not far from 30 000 pounds per square inch.

But it is found, on further investigation, that the load on the structure at the time of the accident was but sufficient to make the maximum

stress on these rods—if properly distributed—20 000 pounds per square inch at the threaded part of the piece; which piece, it has been seen, has been broken by a strain nearly double that figure. The fact is at once inferable that the load came upon these members with such suddenness as to have at least the effect of a live load (as taken in the text-books) and giving a maximum stress equal to twice that produced by the same load gradually applied, *i. e.*, the case in which the load falls, through a height equal to the extension of the piece strained by it, the resistances being assumed to increase directly as the extension up to the point of rupture, —an assumption which is approximately correct for brittle materials like hard cast iron, but quite erroneous in the case of some ductile materials, which latter sometimes give a "work of ultimate resistance" amounting to three-fourths or even five-sixths of the product of maximum resistance by the extension.

This accident was therefore caused by the entrance upon the bridge of a load capable of straining the metal to about one-half of its ultimate strength, if slowly applied, but which, in consequence of its sudden application, doubled that stress.

This sudden action may have been a consequence either of its coming upon the structure at a very high speed, or a result of the loosening of a nut, or of the breaking of a part of either the bridge floor or of one of the trucks of the train. The latter occurrence, permitting the load to fall even a very small distance, would be sufficient.

This paper is not presented as a perfectly satisfactory statement of definite facts from which absolutely reliable conclusions can be drawn. The whole subject is deserving, however, of very careful and very extended experimental investigation, and the writer has been able to obtain but a small amount of satisfactory definite information in regard to it as yet. The figures given do not exactly represent those obtained from any actual case. They do, however, fairly illustrate the limited experience of the writer, and are nearly exact for at least one case; they may serve to indicate the possible value of the cautious application of the method here outlined of studying the causes of such accidents as are considered in the hypothetical case here taken.

The same method may sometimes be used to ascertain the probable cause of a boiler explosion by determining whether the metal has been subjected to overstrain in consequence of overpressure. The causes of accidents to machinery may also be thus detected, and many other applications will suggest themselves to every engineer.

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CLVII.

(Vol. VII.—March, 1878.)

STEAM ENGINE ECONOMY.

A UNIFORM BASIS FOR COMPARISON.

A Paper by CHARLES E. EMERY, M. E., Member of the Society.

In writing a general report on the exhibits referred to the judges of Group XX, Centennial Exhibition, the writer compared the facts available in regard to the economy of steam-engines of various kinds on the uniform basis that the boiler is capable of absorbing 10 000 heat-units per pound of coal consumed. This corresponds to an evaporation of 8.99 pounds of water at 80 pounds pressure, 9.03 pounds at 60 pounds pressure, or, 9.08 pounds at 40 pounds pressure from a temperature of 100° in each case. This evaporation is higher than is usually obtained, but has been so much exceeded in practice* that it is not considered too high for a basis of comparison. The basis, moreover, enables the duty of pumping-engines and other steam machinery to be ascertained and expressed in a very ready and convenient manner. Ten thousand heat-units per pound of coal is equivalent to one million heat-units per 100 pounds of coal, and as the duty of pumping-engines is conventionally expressed in millions of foot-pounds per 100 pounds of coal, it follows on the basis presented that *the number of foot-pounds per heat-unit*

* See examples at page 75 of the report referred to.

represents also the number of millions of foot-pounds duty per 100 *pounds of coal.* The performance of all kinds of steam-engines may be readily compared on this basis. The simplest application is in testing vacuum-pumps, the duty of which may be readily ascertained by noting the height of lift, and the initial and final temperatures of the water lifted. All the heat of the steam not expended in work enters the water, and the work performed lifts the same water. The difference in temperature gives very nearly the number of heat-units imparted to each pound of water lifted, and each pound of water so heated is lifted a certain number of feet high, so the result may be expressed readily in foot-pounds per heat-unit, which, as before stated, equals also, on the basis presented, the number of millions of foot-pounds duty per 100 pounds of coal. For ordinary comparisons the number of millions duty equals the lift divided by the difference between the initial and final temperatures of the water. For more accurate computations, the divisor should be increased by the number of heat-units expended for work per pound of water lifted, which equals the height divided by 772. The height preferably should be calculated from the indications of a pressure-gauge at the bottom of the discharge-pipe, so as to include frictional resistances. If D = duty in foot-pounds per 100 pounds of coal, H = the height of lift per gauge, and t and T = the initial and final temperatures respectively, then

$$D = \frac{1\,000\,000\ H}{T - t + .0013\ H.}$$

Arrangements have been made by the writer to use the same basis in testing pumping-engines, by discharging water from the hot well into the suction of the main pumps, and noting with delicate thermometers the resulting increase of temperature of the water lifted.

A vacuum-pump tested by the writer in 1871 gave a duty, on the above basis, of $4\frac{7}{10}$ millions; one tested by Mr. J. F. Flagg at the Cincinnati Exhibition in 1875, reduced to the same basis, gave a maximum duty of $3\frac{25}{100}$ millions. Several vacuum and steam-pumps tested on this basis at the suggestion of the writer about two years since, gave duties reported as high as 10 000 000 to 11 000 000, the very small steam-pumps doing no better apparently than the vacuum-pumps, which is by no means surprising. Elaborate experiments made with steam-pumps at the American Institute Exhibition of 1867,* showed that average-sized

* See report of Messrs. Holmes, Selden, and Emery, judges, &c., Transactions American Institute, 1867-68.

steam-pumps do not, on the average, utilize more than 50 per cent. of the indicated power in the steam-cylinders, the remainder being absorbed in the friction of the engine, but more particularly in the passage of the water through the pump. Again, all ordinary steam-pumps for miscellaneous uses require that the steam-cylinder shall have 3 to 4 times the area of the water-cylinder to give sufficient power when the steam is accidentally low; hence, as such pumps usually work against the atmospheric pressure, the net or effective pressure forms a small percentage of the total pressure, which, with the large extent of radiating surface exposed and the total absence of expansion, makes the expenditure of steam very large. One pump tested by the writer required 120 pounds weight of steam per indicated horse-power per hour, and it is believed that the cost will rarely fall below 60 pounds; and as only 50 per cent. of the indicated power is utilized, it may be safely stated that ordinary steam-pumps rarely require less than 120 pounds of steam per hour for each horse-power utilized in raising water, equivalent to a duty of only 15 000 000 foot-pounds per 100 pounds of coal on the same basis adopted for the vacuum-pumps. With larger steam-pumps, particularly when they are proportioned for the work to be done, the duty will be materially increased.

Ten thousand heat-units per pound of coal represents an ultimate efficiency of only ($10\,000 \times 100 \div 14\,500 =$) 69 per cent. of the calorific value of anthracite coal, so that ordinarily more than ($100 - 69 =$) 31 per cent. of the heat in the fuel is carried to waste up the chimney. A still greater loss is, however, experienced in utilizing the steam for the purpose of work in the engine. The mechanical equivalent of one heat-unit is 772 foot-pounds, which, on the basis referred to above, corresponds to a duty of 772 millions of foot-pounds per 100 pounds of coal. The most economical steam engines, for instance, pumping-engines of approved types, utilize in the steam-cylinder only about 130 millions, on the same basis, equivalent to an ultimate efficiency of ($130 \times 100 \div 772 =$) 16.84 per cent. of the heat in the steam, and but ($16.84 \times .69 =$) 11.62 per cent. of the calorific value of the fuel. The principal reason for this is that the exhaust steam necessarily carries to waste the heat required to maintain it in a vaporous state at the tension due to the back pressure. This under the most favorable circumstances forms the larger proportion of the total heat of the steam and reduces the opportunities for securing economy within small limits compared with the theo-

retical limit, although the differences between the performances of different engines are great when compared one with another.*

Means for securing economy in steam-engines may be divided into two classes, viz., those of a mechanical nature and those which influence the thermal conditions. As to the first, the necessity of securing tight pistons and valves, ample area of cylinder passages, reduced clearances, etc., are well understood, also the incidental advantages due to a certain degree of compression. Those of the second class act to reduce the cylinder condensation, and include high speeds of revolution, steam superheating, steam-jacketing, and the compounding of engines. High speed of revolution (which does not necessarily imply high piston speed, as generally understood) secures economy, by reducing the time in which the transfers of heat to and from the steam and inclosing walls must take place.†

Superheating the steam has experimentally proved effective for moderate rates of expansion, in which the original temperature required to maintain the gaseous condition of the steam to the point of release was not too high to prevent proper lubrication. Mr. Geo. P. Dixwell, of Boston, Massachusetts, has applied a thermometer to a steam-cylinder, by inspection of which it is possible to regulate the temperature so as to prevent injury to the metal surfaces. The great difficulty is, however, to secure a permanent and reliable superheating apparatus. Steam-jacketing has to a limited extent advantages of the same kind as superheating, and involves no serious difficulties in management. The jackets are most effective on long cylinders of small diameter. In experiments with

* In view of discussions in progress at the date of writing on the proper details of a theoretically perfect steam-engine, it is proper to mention that in the year 1868 the writer designed and partially constructed a non-exhausting experimental engine in which the steam, after expansion in the cylinder, was to be circulated through another vessel, to withdraw the water due to the performance of work; the dry steam was then to be returned to the cylinder and compressed, which it was expected would require less power than the expansion would furnish, and sufficient steam only be received from the boiler to supply that condensed for work. A demonstration of the correctness of the principle only was intended, the power expected being so small that the experimental engine was to be connected to another to keep it in motion. Before the apparatus was completed the funds were diverted to objects of greater immediate necessity, and the subject is mentioned only as indicating the general principle upon which a theoretically perfect steam-engine may be constructed. See description of the apparatus in article on the "Theoretical Steam-Engine," *Scientific American* Supplement, August 18, 1877. See also Prof. Thurston's calculations on a similar subject in *Journal of the Franklin Institute*, October, November and December, 1877.

† The value of this saving was determined by the writer for the Novelty Iron Works, Mr. Horatio Allen, President, in the year 1868, and embodied in a series of tables showing the relative power and economy of different sizes of steam-engines, which tables were afterwards published by Prof. W. P. Trowbridge, the former Vice-President of the company.

United States revenue steamers, hereinafter mentioned, the economy of a steam-jacket on a comparatively short cylinder was found to be 11 to 12 per cent.

Compound engines, in addition to advantages of a mechanical nature, in better distributing the strains and rendering more uniform the rotative efforts, serve also to reduce cylinder condensation by the distribution of the differences of temperature between two cylinders. The radiation to and from the steam and its inclosing walls increases more rapidly than the difference in temperature, so that the aggregate loss, when the difference of temperature is divided between two cylinders, is less than when it all occurs in a single cylinder.* Moreover, the heat imparted to the exhaust steam by the metal of the first cylinder is available for work in the second, and the low-pressure piston acts as a screen between the high temperature in the small cylinder and the low temperature in the condenser.

It is still strenuously denied by many that greater economy can be secured with a compound engine than with a long-stroke single engine, using the same steam pressure. There are coasting steamers of similar size running regularly in the United States using both types of engine, with, it is claimed, substantially the same results; but the boilers for the single engines are evidently the more economical, making an accurate comparison impossible. Strictly comparative experiments have, however, been made by Chief Engineer C. H. Loring, U. S. N., and the writer with engines of different kinds in the steamers of the United States Revenue Marine, and by the writer with some of those of the United States Coast Survey.†

The revenue steamers were of the same size and the boilers very nearly identical. In one steamer was a compound engine with steam-jacketed cylinders; in another, a long-stroke, high-pressure condensing engine (cylinder not jacketed); in another an ordinary low-pressure engine (cylinder not jacketed); and in still another, a high-pressure condensing engine with a jacketed cylinder. The compound engine showed a saving of 12 to 16 per cent. compared with the best performance of either single

* See article by the writer in *American Artizan*, March 8, 1871.

† See article by the writer on "Compound and Non-Compound Engines," *Transactions American Society of Civil Engineers*, vol. iii, p. 68, 1875: *Journal of the Franklin Institute*, February and March, 1875; *Engineering* (London), January, February and March, 1875; *Proceedings of Institution of Civil Engineers* (British), vol. xl., p. 292, and vol. xli, p. 296; also Report of trial of United States Revenue steamer *Gallatin*, *Journal of the Franklin Institute*, February, 1876, and vol. xxi., *Engineering*, 1876.

engine when operated at the same steam pressure. It is believed that substantially the same differences will be found in all cases when equally good engines of both types are compared. The performance of a short-stroke compound engine may be equalled or even excelled by that of a long-stroke single engine, on account simply of the difference in clearance spaces and the superior efficiency of the steam-jacket in the latter case, but by making the compound cylinders in the same form they should still show an advantage. In practice, the economy of marine compound engines is greater than above mentioned, for the reason that the high steam pressure is better maintained with them by the engineers than when single cylinders are used with high rates of expansion, causing difficulties in management.

The following table shows, in line 1 the performance of one of the Leavitt compound beam pumping-engines, at Lawrence, Massachusetts, and in line 2 that of the engines of the *Rush*, one of the revenue steamers previously referred to:

	Approximate Steam Pressure.	Ratio of Expansion.	Diameter of Small Cylinder.	Diameter of Large Cylinder.	Strokes of Pistons.	Revolutions per Minute.	Piston Speed.	Mean Pressure Referred to Large Cylinder.	Indicated Horse-Power.	Water per Indicated Horse-Power per Hour.
			Inches.	Inches.	Inches.		Feet per Minute.	Pounds.		Pounds.
1	90	13.05	18	38	96	16.27	260.3	22.15	196.4	14.02
2	70	6.22	24	38	27	70.84	318.8	24.48	266.6	18.38

The comparison is very interesting. In both engines the larger cylinders are of the same diameter, but the difference in the duty for which the engines were designed required great differences in other proportions and in all the details of construction. In the pumping-engine for use on land there were no restrictions as to weight and space, so a comparatively long stroke could be employed and the connections made through a beam. The marine engine had, however, to be located in a small vessel, and was therefore directly connected and proportioned accordingly. Yet the long-stroke engine was run with so much expansion and at so slow a speed as to develop less power than the smaller one, and the latter was less economical, on account of the lower steam pressure and rate of

expansion and the relatively greater proportion of waste room in the cylinder, incident to the necessary use of ordinary slide-valves. The engine of the *Rush* was, however, more economical than the ordinary stationary compound engines used for manufacturing purposes, as the latter, according to published reports in the engineering journals, require the evaporation of not less than 20 pounds of water for each indicated horse-power. The Lawrence engine contains all well known means for securing maximum economy of steam, and it is probable that few if any engines are working with greater economy in respect to the indicated power. The performance is, however, much below that given by calculation when all the conditions are taken into consideration, other than the slight distortion of the theoretical indicator diagram found in practice and the important loss due to cylinder condensation.

In an engine using a total pressure of (90 + 14.7 =) 104.7 pounds, expanded 13.5 times in a cylinder, with clearances, etc., equal to .02 of the displacement, the calculated cost of one horse-power per hour, or 1 980-000 foot-pounds, should be only 8.12 pounds of water evaporated from the initial pressure, on the basis that the curve of expansion is hyperbolic, and that the consumption of steam equals the volume at the initial pressure, required to fill the cylinder to the point of suppression, plus that condensed for the total work. With a pressure of 100 pounds above the atmosphere, and an expansion of 20 times, there should be required on same basis the evaporation of only 6.00 pounds of water per indicated horse-power per hour. It is probable that the practical results obtained with the latter pressure and expansion would be little or no better than those from the Lawrence engine, on account of the greater cylinder condensation due to the increased expansion.

The above calculated performances, and the practical results obtained with engines and other steam machinery of various kinds, is shown in the accompanying table, in connection with the relative efficiencies obtained by considering the heat-units in the steam and the calorific value of the fuel. The table and a portion of the above are from the report previously mentioned, and the references are to pages therein.

Number of Line for Reference.	1	2	3	4	5	6	7	8
	Description.	Steam Pressure.	Ratio of Expansion.	Water evaporated per indicated Horse-Power per Hour.	Comparative Results on Basis that 10,000 Heat-Units are imparted to Water per Pound of Coal. See pages 21 and 115.** Calculations based on a Temperature of Feed of 100°. Duty in Millions of Foot-Pounds, per Indicator, per 100 Pounds of Coal. Pages 21 and 115.**	Relative Efficiency, based on the Proportion utilized of the total Heat of the Steam.	Relative Efficiency, based on the Proportion utilized of the Calorific Value of Anthracite Coal.	Net Effective Duty for lifting Water, in Millions of Foot-Pounds, per 100 Pounds of Coal.
1	Calculated performance. Maximum	.....	.....	.....	772.0	1.00	.69	
2	Calculated performance (see page 120)**	100	20	6.005	295.2	.382	.264	
3	Calculated performance	90	13.5	8.122	218.6	.283	.195	
4	Lawrence compound beam pumping-engines	89.4	13.7	14.019	126.7	.164	.113	
5	U. S. Revenue steamer Rush,* compound engine	69.2	6.22	18.384	97.03	.126	.087	
6	U. S. Revenue steamer Gallatin,* vertical cylinder with steam-jacket	67.2	4.19	21.48	83.08	.108	.074	
7	U. S. Revenue steamer Dexter,* vertical cylinder without steam-jacket	67.1	3.49	23.905	74.66	.097	.067	
8	U. S. Revenue steamer Dallas,* vertical cylinder without steam-jacket	32.0	3.13	26.945	66.91	.087	.060	
9	U. S. steamer Mackinaw,† inclined cylinder without jacket	49.0	2.2	30.306	59.16	.077	.053	
10	U. S. steamer Mackinaw, steam superheated	52.0	3.2	22.725	78.83	.102	.070	
11	Non-condensing engine, with governor cut off,‡ (st. jacket)	81.7	5.0	25.482	69.81	.090	.062	
12	Non-condensing engines, regulated by throttle	.....	.....	.....	30 to 45	.04 to .06	.03 to .04	
13	Pumping-engines							30 to 110
14	Steam-pumps. Large size proportioned for the work to be done							15 to 30
15	Steam-pumps. Small sizes for ordinary uses. See page 22**							8 to 15
16	Vacuum-pumps. See page 21,** also page 61 herein							3 to 10
17	Injectors when used for lifting water not required to be heated. See page 96**							2 to 5

* See references in foot-note, page 119,** also†, page 64 herein.

† See vol. ii. Isherwood's *Experimental Researches in Steam Engineering*, pp. 77–116.

‡ *American Institute Reports*, 1869–70, 1870–71.

** General Report of the Judges of Group XX, Philadelphia Int. Exhibition. Lippincott & Co., Philadelphia.

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CLVIII.

(Vol. VII.—March, 1878.)

THE INCLINE PLANE RAILROAD AT MADISON, IND.

ITS HISTORY AND OPERATION.

By M. J. BECKER, C.E., Member of the Society.

The history of the Incline Plane Railroad at Madison, Indiana, the changes in the methods of its operation, and the information which may be drawn from the experience of its working during a period of more than thirty years seem to present matter of considerable interest to engineers. I have been able to secure the following account, which has been mainly collected by Mr. R. Wells, Master Mechanic of the Jeffersonville, Madison and Indianapolis Railroad, and which contains all the reliable information it has been practicable to gather on the subject.

The Incline Plane at Madison was constructed under the supervision of Thomas Morris, as Chief Engineer, and was part of the main line of road between Madison and Indianapolis. The time when the Plane was first opened for traffic has not been ascertained, but business on it was carried on by using horses to draw the cars up until the fall of the year 1847.

The foot of the incline plane is 3620 feet from the depot in Madison (the terminus of the road). This track, from the depot to the foot of the incline has a grade averaging 1 in 110, and for about 800 feet next the

foot of the incline is on a curve of about 1000 feet radius. The incline is 7040 feet long, and the grade averages 1 in 17, but for about half the distance is 1 in 16.5, and at two places is a little heavier than that from the settling of the embankment. The track was originally straight, but owing to the sliding of the embankment at a point near the centre of the incline, it was slightly curved at that place. The gauge of the track was originally the same as it is now, 4 feet 8½ inches. In the year 1845, M. W. Baldwin, of Philadelphia, built a locomotive called the De Witt Clinton, having six coupled driving wheels 42 inches in diameter and cylinders 13 × 20 inches, for working this incline; but as the engine could draw but two cars up at a time, depending on its weight on the rails for adhesion, it did not prove satisfactory, and as stated before, horses were used to draw the cars up the incline until the year 1847. About the year 1845, an old gentleman named Hoyt, who lived near Dupont, on the line of the road, fourteen miles from Madison, conceived the idea of laying a cast iron rack on the outside of the rails of the track, and level with the top of the rails, to be laid on steep inclines such as this, and a wheel, or segments with cogs corresponding to those in the rack was to be attached to the outside of one of the driving wheels so that when the locomotive came to the part of the line where the rack was laid, the cogs on the one would work into those of the other, preventing slippage, the tire of the wheel at the same time carrying its weight the same as other places. To obtain the requisite power to ascend inclines, Hoyt proposed to have a pinion wheel arranged so as to gear into the wheel attached to the driving wheel, driving the pinion by a cylinder and crank which, when not needed, was to be thrown out of gear. Hoyt had a rough model of his plan and explained it to the officials of the road in the various interviews he had with them from time to time, and among others, to Mr. A. Cathcart, who had charge of the machinery of the road at that time. It seems that Hoyt's plan was not considered practicable, and he met with but little encouragement. but Cathcart, soon after, perfected his own plan which he carried out and put in practice in the fall of the year 1847. He evidently was indebted to Hoyt for the original plan. Yet he used but little of Hoyt's plan, except a rack and a wheel to work in it. As a settlement for any claims that Hoyt might have to any part of Cathcart's arrangement, the latter paid the former $1000 to relinquish all claims he might have in the invention used.

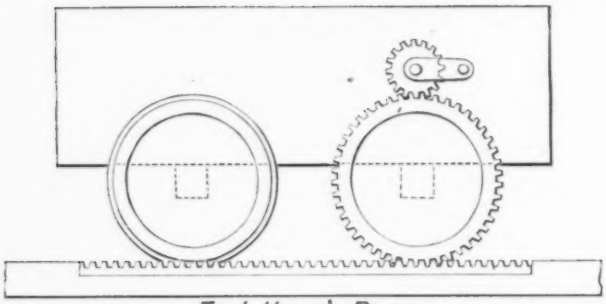

FIG. 1. HOYT'S PLAN.

Hoyt's plan is represented in Fig. 1. It is a copy of his original sketch.

Cathcart's plan is represented in Fig. 2. His locomotive had eight driving wheels coupled in the usual manner, 42 inches in diameter, and a pair of inclined cylinders 16 inches diameter and 20 inches stroke attached to the side of the smoke box, as shown in the sketch. In addition to this the engine had a pair of vertical cylinders 15 inches diameter and 18 inches stroke, shown at *B*, Fig 2. These cylinders were attached to a shaft held in place by a frame connected with the vertical cylinders, and also the frame of the locomotive. On the centre of this shaft was a pinion 16 inches in diameter, shown at *C*, gearing into another pinion shown at *D*, which, when in use, geared into the rack laid in the centre of the track as shown at *E* and *F*, Fig. 2. This pinion, *D*, was 32 inches in diameter, and was movable vertically in curved jaws, the radius of this curvature being the distance from the centre of the pinion *C* to that of the pinion *D*. For the purpose of raising the pinion *D* out of the rack when not in use, and also for holding it down in place to prevent it from slipping out of gear while at work, a cylinder of 8 inches in diameter and 12 inches stroke, shown at *G*, Fig. 2, was used by means of an L crank and suitable rods and connections, the force of the steam on one side of the piston holding the pinion down in place, and when changed to the other side raising it up out of gear where it was held by a pawl until it was to be lowered again for use. The cogs were 4 inches pitch and 8 inches face on the rack, and 8 inches face on the pinion wheel. The top of the cogs in the rack was 6 inches above the top of the rails of the track. This rack was made in sections of about 3 feet in length. It was 8 inches wide at the bottom, where it rested on the timber, as shown

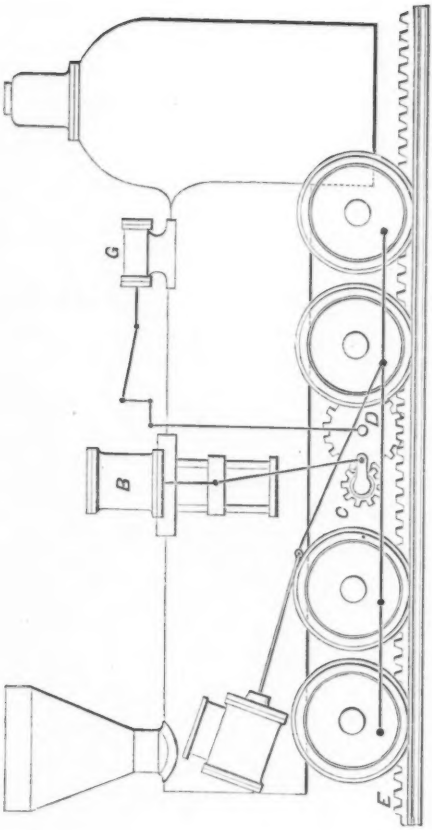

FIG. 2. CATHCART'S PLAN.

by Fig. 3, there being a flange on each side of 1 inch on which the head of the spikes rested that held the track down to the timber on which it was laid. These flanges were notched at proper intervals of ½ inch deep for the spikes, as shown at *R*, and there were also projections, *S S*, on the bottom which were let into the timber to prevent their being moved out of place. The timbers upon which the rack rested were 6 inches deep by 12 inches wide, laid on the cross-ties in the centre of the track, the ties being notched about half an inch deep to receive them, as shown in Fig. 3, at *T*. The rack was made from cold blast iron (cast). The cogs were not chilled. The pinion wheels on the locomotive were made of cold blast iron, and made in chills so as to chill the cogs to prevent wear as far as possible.

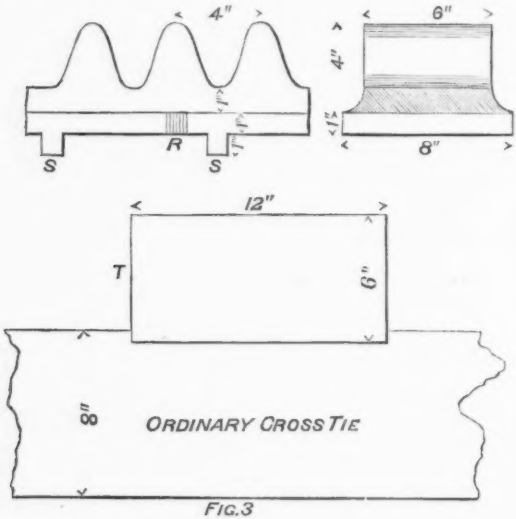

FIG. 3

This arrangement, known as Cathcart's plan, was complete and put in operation in the fall of the year 1847, and was continued in use until July, 1868, giving general satisfaction. The heavy business of the road from 1849 to 1854, and the lighter business afterwards, was all done on this plan with the two locomotives of the Cathcart plan, although the speed at which trains could be taken up and freight cars taken down was comparatively slow; twenty minutes for passenger trains and twenty-

five minutes for freight trains was about the time occupied in ascending the incline of 7 000 feet with full trains, and the expense in maintenance of the rack and locomotive was heavy. The breakage of castings of the rack and the wearing out of pinion wheels of the locomotive, and cost of keeping up its complicated parts, the decay and removal of timber under the rack were all sources of trouble and expense, amounting to nearly the same whether heavy or light trains were drawn, and amounting to more than the business of the road would warrant in the last two or three years that arrangement was used. Owing to the expense of operating the incline, and the slow speed at which only it could be worked on this plan, it was determined to try the adhesion plan, by the use of very heavy and powerful locomotives, depending alone on the weight of the wheels on the rails for their adhesion. Consequently one was built in the company's shop of a pattern represented in Fig. 4, and put in service in July, 1868, since which time the Cathcart plan has been abandoned, and the plane worked entirely on the adhesion plan, which has been found to be much cheaper, quicker, and equally as safe, and more reliable. The principal dimensions of the locomotive, represented in Fig. 4, are as follows: Cylinder 20½ inches diameter, 24 inches stroke; driving wheels (five pairs), 42 inches diameter; boiler, 56 inches diameter, $\frac{7}{16}$ thick, 201 two-inch tubes 12 feet long; fire-box, 5 feet 3 inches long, 5 feet 8 inches deep, 4 feet wide at top and 3 feet at grate, has 116 square feet fire-box heating surface, and 1 262 feet of tube surface. It is what is commonly called a tank locomotive, carrying wood (which is used for fuel) and water all on the five pairs of coupled wheels. The water is carried in two long cylinders (one on each side), shown at *H H*, connected to a centre water tank at the back, *I I*. These cylinders are flattened on the side next the boiler, as shown in the end view at *J*, but are the full circle back of the boiler. The two and the centre tank at the back end hold 1 800 gallons of water. The total weight of the locomotive, with fuel and water, is 112 000 pounds. A steam brake is situated at *K*, by which one-third of the weight of the locomotive can be made to rest on slides or runners forced down on the rails; friction brakes are also attached to the drivers, and applied by means of a screw sufficiently powerful to hold the locomotive on the incline, but are not used in ordinary service.

All freight trains are let down the incline by the locomotive using compression in the cylinders as a means of retarding power. For in-

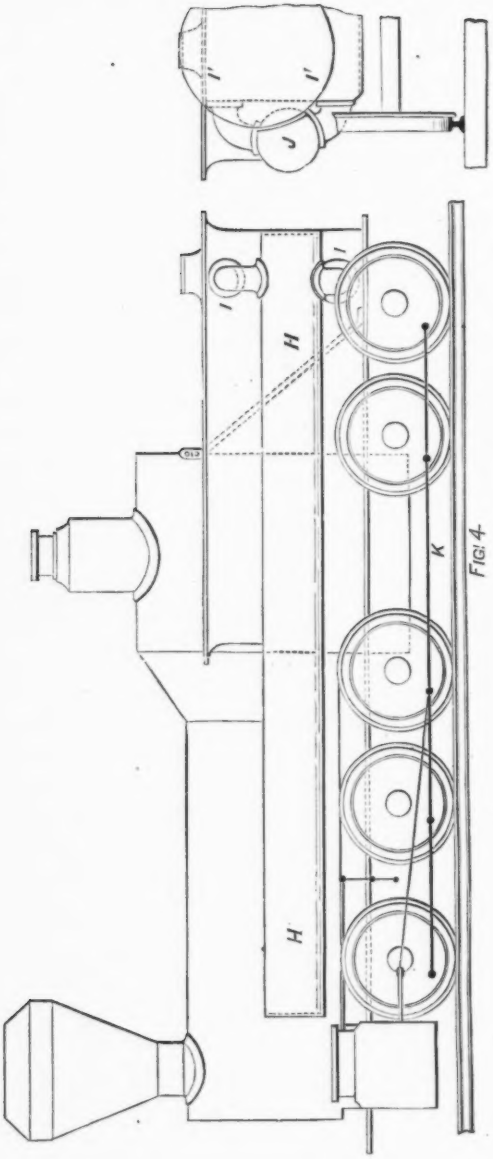

FIG. 4.

stance, when backing down the incline the valves of the cylinders are moved by the forward motion which if there was no leakage of the pistons would either force air into the boiler or cause the wheels to stop turning in that direction. But to regulate the compression so as to give the required retarding force, the two ends of the cylinders are connected by a 1-inch pipe, in which is a sliding valve closing and opening to the desired size, permitting the compressed air on that side of the piston to flow through to the opposite side of it, and being regulated for both cylinders by one lever and to any speed descending to ten miles per hour.

The loads drawn up the incline and the time required may be explained by the record of one of the number of trips made April 9th and 10th, 1875, to test the quantity of fuel used, adhesion to rails, etc. :

Trip No. 1 was made on a dry day, temperature of the atmosphere 70° Fah. The load was eight cars loaded with coal.

Weight of cars and coal	308 360 lbs.
" " locomotive	112 000 "
Total	420 360 "

Speed at foot of incline, 2 miles per hour.

Time of leaving foot of incline—

At	3.40	P. M. ;	steam pressure in boiler,		140 lbs.
"	3.45	"	"	"	140 "
"	3.50	"	"	"	143 "
"	3.52	"	"	"	140 "

Arrived at top of incline at 3.53 P. M. ; steam pressure in boiler, 130 lbs.

Distance run, 7 040 feet. Time, 13 minutes. Fuel used, $\frac{6}{10}$ of one cord of dry beech wood. No sand was used, and no slippage of the drivers occurred. The speed averaged 6 miles per hour. Trip No. 4 was made with two passenger cars, weighing 88 000 lbs. Locomotive, 112 000. Total, 200,000 lbs. Speed when going on the foot of the incline, 6 miles per hour. Time running the 7 040 feet, 5½ minutes=14½ miles per hour. Steam pressure in boiler, 132 pounds. Fuel consumed, $\frac{3}{10}$ of a cord of dry beech wood. No slippage of the drivers at any time, and no sand was used. The trip was made at 7.35 in the morning. No difficulty has been experienced at any time in working this incline for want of adhesion, either in winter or summer, dry or wet weather, and this plan has proved much more satisfactory than the one previously

used, and no accident to the locomotive has ever happened while ascending or descending that in the least interfered with safety.

It may not be without interest to the parties desiring information on this subject to know that a patent was granted to one Emor Rimber, of Rimberton, Chester County, Pa., March 5th, in the year 1831, the original papers coming into my hands while making inquiry on this subject.

The paper is signed by Andrew Jackson, who was President of the United States at that time, and Martin Van Buren (afterwards President), Secretary of State.

The sketch shown at Fig. 5 will explain Rimber's plan, and is a reduced copy of the original accompanying the patent documents.

As will be seen, Rimber proposed the use of a rack and pinion in the centre of the track, carrying the driving wheels of the locomotive (one pair of them) on a pair of smaller wheels of a diameter corresponding to that of the pinion wheel in the centre. The track upon which these small wheels run was elevated sufficiently to raise the driving wheels clear of the track, permitting them to turn in conformity to the smaller wheels and the pinion wheel working in the rack, the power being applied by the cylinders and crank on the outside of the driving wheels, as when running on an ordinary track. A "drag" was proposed to follow along the rack, so as to prevent running down the grade in case of accident to the machinery in ascending the incline, as shown in the sketch. As far as I can learn, Rimber's plan was never put in operation.

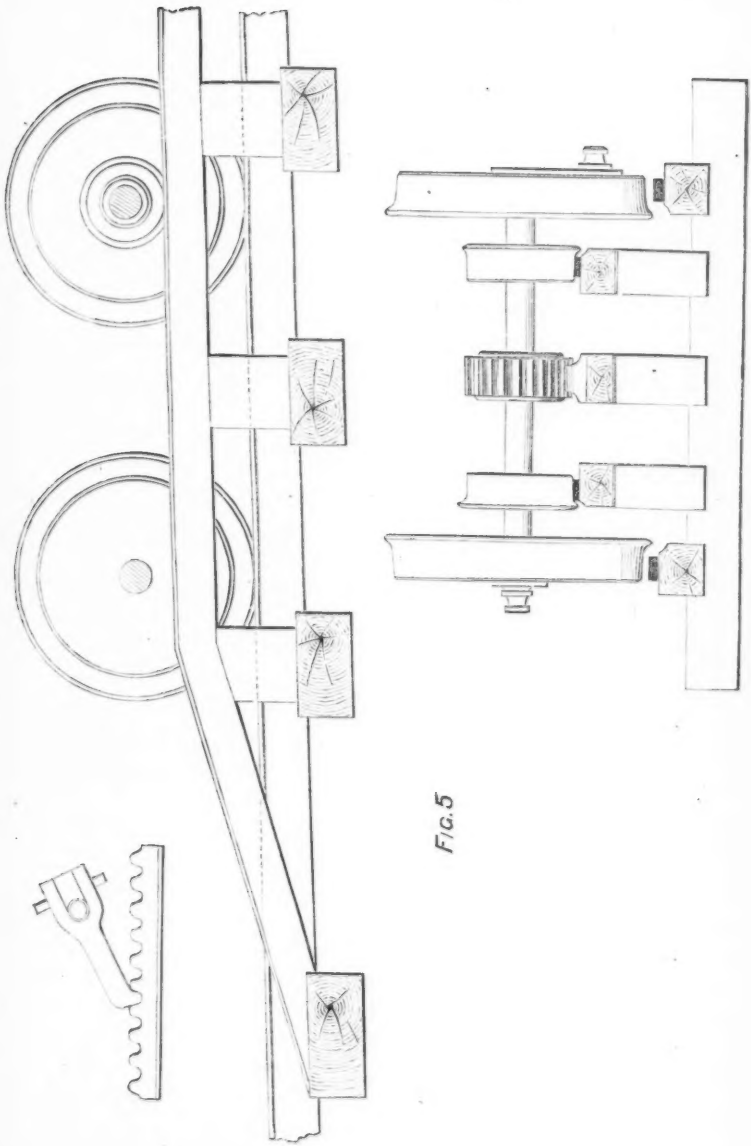

FIG. 5

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CLIX.

(Vol. VII.—April, 1878.)

ON THE THEORETICAL RESISTANCE OF RAILROAD CURVES.

By S. WHINERY, C. E., Junior of the Society.

The writer is impressed with the belief that too little attention has been given to the theory of the resistance which a curved track offers to the passage of a railway train. Efforts have often been made to determine the amount of this resistance experimentally, with greatly varying results, but very few attempts have been made, so far as the writer is aware, to investigate analytically the elements which together make up the sum of such resistance.

While it is probable that in the end, we shall be compelled to depend upon experiment for accurate information on this point, it will be admitted that intelligent theory is greatly needed to assist the experimenter in determining the amount and character of the resistance in question, and the inventor in devising means to surmount each of the elements which together make up the whole. It is one of the cases where analytical investigation must go hand in hand with careful experiment, if accurate and intelligent results in detail are to be reached.

As a contribution toward such investigation the following is submitted :

§ 1. When a railway car with cylindrical wheels of the same diameter moves along a track which is straight and transversely level, impelled by a power, which we shall call the tractive power, acting parallel to the rails, there is no tendency developed to move laterally or to press against either rail ; for the only force which acts upon it, other than the tractive power, is that of gravity, which is applied perpendicular to the plane of the supports, and consequently develops no tendency sidewise.

But when the direction of the forward motion is changed by a curved track, another force in addition to the two named is brought into action. This is the centrifugal force, which is expressed by the well known formula :

$$E = W\frac{V^2}{32.2R}$$

where F = the centrifugal force,
W = weight of the moving body,
V = velocity in feet per second
and R = the radius of the curve in feet.

We may consider the force of gravity and the centrifugal force as both applied to the centre of gravity of the car in a plane perpendicular to the tangent to the curve at that point. The force of gravity acts vertically downward, and the centrifugal force acts horizontally outward from the centre of the curve, and the resultant of the two is represented in direction and amount by the diagonal of a rectangle of which one side equals the force of gravity and the other the centrifugal force. But the former $= W$, and the latter $= W\frac{V^2}{32.2\,R}$; therefore this resultant is represented by the expression :

$$\sqrt{W^2 + \left(\frac{WV^2}{32.2R}\right)^2} = W\sqrt{1 + \frac{V^4}{1036.84R^2}}. \qquad (1.)$$

If we call α the angle which the resultant makes with the vertical (W), we shall have :

$$\sin.\ \alpha = \frac{\frac{WV^2}{32.2R}}{W\sqrt{1 + \frac{V^4}{1036.84R^2}}}, = \frac{\frac{V^2}{32.2R}}{\sqrt{1 + \frac{V^4}{1036.84R^2}}}$$

The action of this resultant develops a tendency to press the wheels against the outer rail of the track. To counteract this tendency we may incline the plane of the rails until it shall be perpendicular to the resultant; that is, we must make the angle between the plane of the rails and the plane of the horizon $= \alpha$.

Calling g the distance from centre to centre of rails,* the elevation E of the top of one rail above the other is found by the formula $E = g \sin. \alpha$.

If we put for g its ordinary value, $4' 8\frac{1}{2}'' + 2\frac{1}{4}'' = 4' 10\frac{3}{4}'' = 4.896$ feet and the expression found above for the value of sin. α we shall have:

$$E = 4.896 \frac{\frac{V^2}{32.2R}}{\sqrt{1 + \frac{V^4}{1036.84R^2}}} \qquad (2.)$$

Table 1 gives values of E for various curves and speeds, calculated by the above formula.

It will be noticed that the values of E here given, differ somewhat from those usually given in tables of this kind. (See, for instance, Henck's Pocket Book, p. 120.) This difference is to be attributed, 1st. To the different value of g used, as explained above. 2d. Examination will show that other tables are based on the formula $E = g$ tan. α, which is clearly erroneous. Recurring to our rectangle of forces we find that

$$\text{tan. } \alpha = \frac{W \frac{V^2}{32.2\,R}}{W} = \frac{V^2}{32.2R}$$

which expression multiplied by g is the formula usually employed.† At ordinary speed the results do not materially differ, but the erroneous character of the usual one becomes evident when the speed is made very great. When the "coming train" shall glide around a 12° curve at the rate of 90 miles per hour, the common formula would require, for a track of 4.7 feet gauge, a superelevation of the outer rail of more than five feet—a requirement, to comply with which would sorely puzzle the intelligent Hibernian section boss of the future. The difference due to this cause is less apparent in the above table, as compared with others, because the greater value of g used, tends to neutralize its effect.

* The practice of calling g equal to the distance from inner face to face of rails, that is, equal to the guage of the track, is manifestly erroneous, unless care is taken to measure the superelevation of the outer rail from those points, and not, as is the usual custom, from the highest point of each rail.

† Henck, p. 89.

TABLE No. 1.

Elevation of outer above inner rail for various curves and speeds. Gauge of track 4 ft. 8½ in.

DEGREE OF CURVE.	SPEED IN MILES PER HOUR.							
	10	15	20	25	30	40	50	60
1°	0.006	0.013	0.023	0.036	0.051	0.091	0.143	0.205
2°	0.011	0.026	0.046	0.071	0.103	0.183	0.285	0.410
3°	0.017	0.038	0.068	0.107	0.154	0.274	0.427	0.612
4°	0.023	0.051	0.091	0.143	0.205	0.364	0.570	0.810
5°	0.029	0.064	0.114	0.178	0.256	0.455	0.704	1.005
6°	0.034	0.077	0.137	0.214	0.308	0.545	0.843	1.196
7°	0.040	0.090	0.160	0.249	0.358	0.634	0.977	1.379
8°	0.046	0.103	0.182	0.285	0.409	0.739	1.111	1.557
9°	0.051	0.116	0.205	0.320	0.460	0.810	1.242	1.734
10°	0.057	0.128	0.228	0.356	0.511	0.898	1.376	1.895
11°	0.063	0.141	0.250	0.390	0.560	0.982	1.492	2.049
12°	0.068	0.154	0.273	0.426	0.611	1.068	1.615	2.200

§ 2. As stated above, the resultant of the weight and the centrifugal force is represented in magnitude by the expression, formula (1).

It is evident that, when this resultant is applied normal to the plane of the track, it, and not W represents the true pressure on the rails. Calling this modified pressure W', we shall have

$$W' = W\sqrt{1 + \frac{V^4}{1036.84R^2}}. \qquad (3.)$$

§ 3. To effect the required difference of elevation between the outer and inner rail, it is the general practice to superelevate the outer rail above the general level of the track. This practice is, to say the least, doubtful. The moment a car strikes a curved track its centre of gravity is acted on by the centrifugal force which tends to overturn the car outward on the outer rail, but if at the same time the support of the inner wheel is removed or depressed, the weight of the car will tend to turn it on the outer rail inward. Thus the two forces acting in opposite directions will tend to neutralize each other. Another consideration which does not seem to have received attention in practice, is the fact that where a curve begins on an ascending gradient, the superelevation of the outer rail materially increases the steepness of the grade at that point. Thus if, on an ascending grade of forty feet per mile, a superelevation of the outer rail, of 0.8 feet is acquired in a distance of two hundred feet, the average gradient for that distance is increased to over fifty feet per mile.

There is a practical difficulty in securing, on circular curves, the necessary difference in elevation of the rails at exactly the proper time. Theoretically, the whole change of level should take place simultaneously with the beginning of the curve, a condition obviously impossible to comply with. But this difficulty may be overcome by beginning and ending the curve with a parabola whose radius, beginning infinitely great, shall become equal to the radius of the circle into which it merges. The change of level of the rails would thus begin with the beginning of the curve, and gradually increase until its maximum was reached at the beginning of the circle.

It is safe to predict that, in the future, when fabulous train speeds become possible, through perfection of track, all curves will begin and end with the parabola.

§ 4. If a cylindrical wheel is placed upon a plane surface and compelled to roll in a circular track, with its axis pointing continually toward the centre of the circle, there will be, in addition to the forward

motion in the prescribed path, a motion of revolution around the centre of the line of contact between the wheel and the plane. This motion is illustrated by the twirling of a coin placed on its edge on a table. When the wheel completes one revolution in its circular path it will have turned one complete revolution on its line of contact with the plane. This turning motion develops a certain amount of frictional resistance to the forward progress of the wheel, which we wish to determine.

The surface of contact between a cylinder and a plane is theoretically a line, but in reality it is doubtless a very narrow parallelogram. If weight is placed upon this parallelogram and it is then revolved around its centre, *i. e.*, the intersection of its diagonals, sliding on the plane upon which it rests, we shall have for the work done in overcoming the friction for one entire revolution,

$$n = \frac{l}{3}\pi W f.$$

Where n = the work done,

l = the length of the parallelogram,

W = the weight resting on the parallelogram,

f = the co-efficient of friction between the surfaces in contact.

The length of the line or parallelogram of contact between a car wheel and the rail is found to be about one inch = 0.0833 feet in ordinary practice. The value of the co-efficient of friction between chilled cast-iron wheels and iron or steel rails, varies with the velocity and the varying conditions of practice, from 0.15 to 0.25; for the purposes of this paper it will be sufficiently accurate to call it 0.2. Substituting these values of l and f in the above equation, we shall have

$$n = .017453\, W.$$

Dividing this by 360 we shall have the work done for one degree of angular change of direction in the motion, and dividing this again by the length of the arc of one degree in the circular path of the wheel, which in the case of railroad curves we may call $\frac{100}{D}$ (D = degree of the curve), we shall find the measure of this element of resistance for any curve to be

$$n = \frac{.017453\, W D}{36000} = .000000485\, W D. \qquad (4.)$$

This expression represents, as we shall see, an insignificant part of the total curve resistance, and would not be worthy of consideration were it not that we wish to determine, not alone the total amount of curve resistance, but also the relative magnitude of each element.

§ 5. When a pair of cylindrical wheels, of equal diameter, fixed immovably to the same axle, roll around a curved track, with the axle pointing continuously to the centre of the curve, it is evident that one of the wheels must slip on the rail over a distance equal to the difference in length between the inner and outer rail. If the track describes a complete circle, the difference in length of the two rails, whatever may be the radius of the curve, $=2g\times\pi$, and the difference for one degree of angular motion $=\frac{2g\times\pi}{360}=\frac{g\pi}{180}$. If now we multiply this expression by $\frac{W}{2}$ (since one-half the load rests on the slipping wheel), and by the co-efficient of sliding friction between the rails and wheel f, we shall have the total work done in thus sliding one wheel over the arc of one degree of the curved track, and if we divide this by the length in feet of that arc $\frac{100}{D}$, we shall have as the measure of this element of resistance S,

$$S=\frac{g\pi WDf}{100\times360}=\frac{4.896\times3.1416\times0.2\,WD}{36000}$$

$$=0.00008545\,WD. \qquad (5.)$$

§ 6. In practice, two or more pairs of wheels are so fixed in a rigid frame, that the axles are held parallel to each other. When such a system is placed on a curved track it is impossible for it to conform readily to the curve, or for the axles of each pair to point towards the centre of the curve; but if it rests symmetrically on the rails, the radius of the curve to which the axles are parallel, will pass midway between the first and last axles of the system. If motion be given to the system it will tend to move in the direction of the tangent to the curve at the extremity of that radius. But the rails pressing against the flanges of the wheels will compel it to follow the curve. The pressure thus exerted, develops a resistance to the onward motion of the body, which may be approximately determined.

If we denote by m the distance from the point where the rail presses against the flange of the wheel in guiding it along the curve, back to the radius of the curve, which passes through the centre of the truck, in a line perpendicular to that radius, and denote by β the angle which measures the change of direction of the curve in that distance, we shall have sin. $\beta=\frac{m}{R}$

It is obvious that this angle remains constant while the car passes around the whole length of any curve of constant radius.

In the effort to follow the straight line as described above, the outside forward wheel of the truck tends to roll diagonally across the face of the rails. Thus, Fig. 1, any given plane in the wheel, as AD, would, if unrestrained, roll in the direction AB: but the resistance of the rail, against the flange causes the point of contact A, to follow AC. To do this it is obvious that the wheel with the weight resting upon it, must be slipped on the rail a distance BC, $= AC \times \sin. \beta$. Calling AC unity, and assuming 0.2 as the co-efficient of friction, the resistance offered to the onward motion of the car will $= W \times 0.2 \times \sin. \beta$ (6). But while this expression represents the amount of work done by the rail in compelling the wheel to follow the curve, it does not represent the whole of the power expended in the operation; for in the act of revolving the wheel is moved continuously at the point of contact between the flange and the rail. The pressure at this point is evidently $W \times 0.2 \times \sin. \beta$ and this pressure will cause a frictional resistance $= W \times 0.2 \times \sin. \beta. \times 0.2 = 0.04\ W \times \sin. \beta$. But the tractive power acts on this frictional resistance through the agency of a lever of which the long arm is the radius of the wheel, the fulcrum, the point where the tread of the wheel rests on the rail, and the short arm, the distance from the latter point to the point of contact between the flange and the rail. The last named distance is of uncertain magnitude, being dependent on the size of the wheel and the form of the flange. It is probable that in practice it does not vary much from four inches. Assuming that as the correct value, and calling the radius of the wheel sixteen inches, we shall have for the tractive power necessary to overcome this particular element of resistance, $\frac{4\ (.04\ W \times \sin.\ \beta)}{16} = 0.01\ W \times \sin.\ \beta$, and for the whole resistance due to compelling the wheel to follow the curve, which we may call p,

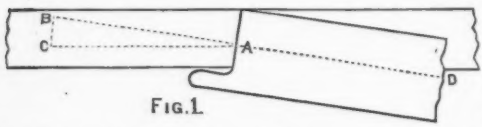

FIG. 1.

$$p = 0.2\ W \sin.\ \beta + 0.01\ W \sin.\ \beta, = 0.21\ W \sin.\ \beta. \qquad (7.)$$

The conditions above described strictly apply to only the outer front and inner rear wheels of the truck. But a little consideration will show that the four wheels of the truck are together subject to just four times

the resistance for one wheel, as given above; for the inner front and outer rear wheels must be constantly slipped toward their respective rails in order to conform to the curve, to the same extent as the other two wheels are slipped in the same directions; but the whole of the work is done by the flanges of the outer front and the inner rear wheels.

§ 7. There is another element which may affect the results given by the above formula, which may be thus stated. In considering the element of the friction between the wheel flange and the rail, we have, in effect assumed that the side of the rail is vertical; that is, that a point of the flange in contact with the rail moved downward in a plane perpendicular to the top of the rail and the tread of the wheel. If this is not the case, any given point of contact of the flange must slide down an inclined plane, thus resolving the power applied into two components, only one of which is available for the work required. While the energy thus lost is insignificant in amount, in the ordinary practice, it is nevertheless a source of resistance, and unless there are weightier reasons for a different practice, the sides of rails should be made vertical, and the flanges of wheels so shaped that the rail will retain this form under wear.

§ 8. When individual cars are joined into a train and drawn by a locomotive along a curved track, the line of tractive power or tension forms a funicular polygon, of which each car forms a side, and whose angles are located at the points where the cars are coupled together. At each one of these angles a certain amount of the tractive power or energy is absorbed and the remainder transmitted to the following side of the polygon. In other words, at each angle the tractive power is decomposed into two components, one of which is expended in drawing the following car along the track, and the other tends to move the angle itself towards the centre of the polygon, which is identical with the centre of the curve of the track.

If we denote by φ the acute angle between the longitudinal axes of two adjoining cars, and by T the tractive power exerted by the forward car, and by T' the tractive power transmitted to the following car, we shall have $T' = T \cos. \varphi$. (8), and the amount of power lost at the angle will be $T - T \cos. \varphi = T (1 - \cos. \varphi)$. (9). If we know the power which, applied to a single car in the direction of its longitudinal axis will move it along a curved track under given conditions, we may, by the aid of formulæ (8) and (9) readily determine the aggregate amount of energy thus lost in any train.

The second component into which T is decomposed, tends to move the angle of the polygon toward its centre. Its practical effect, were it of sufficient magnitude, would be to press the flange of the inner wheels against the rail. Now the polygon rests on the track at as many points as there are trucks in the train; that is, at two points in each car, and as the line of tractive power must be considered as straight from one point of application to another, we may consider the polygon thus formed as having double the number of sides heretofore considered, and the acute angle between adjacent sides will be $\frac{\varphi}{2}$, and the radial component, a, of the tractive power will be, nearly,

$$a = T \sin. \frac{\varphi}{2} \qquad (10)$$

It would seem at first thought that this component ought to relieve, to some extent, the pressure against the outer rail expressed by formula (6), but it will be remembered that the inner rear wheel of the truck presses against the inner rail with the same force that the outer front wheel presses against the outer rail in causing the truck to follow the curve. Hence, this component can be of no service in diminishing the sum of the resistance due to that cause.

§ 9. We have purposely neglected so far to take into account the element of coned wheels in modifying the resistance due to curved track, preferring to consider the whole subject briefly in a separate section. There is, perhaps, no one thing in railroad practice upon which there is such diversity of opinion, as upon the subject of the utility of coning the wheels of cars. It is needless to here enumerate the various shapes which such opinion takes. It is certain that the tendency of the day is to reduce the amount of coning to a very small quantity, and many boldly predict that coned wheels will soon be a thing of the past.

We shall use, for the purposes of illustration in this paper, the standard wheel of the Pennsylvania Railroad, which is 33 inches in diameter, has a tread of 4½ inches, and a coning of ½ inch; that is, the diameter of the wheel at its outer edge is ½ inch less than at the tangent point of the curve which joins the tread and the flange. The space allowed between each rail and the flange is one-half inch, and, therefore, the total lateral "play" of the wheels on the track is limited to one inch. When a pair of such wheels are resting symetrically on the track they are of the same diameter at the points of support. But if moved toward one side so that the flange of one wheel is in contact with the rail the

diameter of that wheel will clearly be increased an amount $= \frac{\frac{3}{8}'' \times \frac{1}{4}''}{4\frac{3}{8}''} = \frac{3''}{70}$, and as the diameter of the other wheel will be decreased a like amount, the difference of their diameters in the new position will be $\frac{3''}{35}$; and if the pair of wheels in this position were put in motion they would, if unobstructed, describe a circle whose radius reckoning from the centre of the track, and calling the diameter of the lesser wheel 33″, we shall find to be 1887 feet, which is the radius of a 3° 8′ curve. Therefore, if the wheels were placed on a curved track of 1887 feet radius with the flange of the wheel in contact with the outer rail, they would follow the curve without any constraining influence by the rails, and consequently their forward motion would be entirely free from the frictional resistance expressed by formulæ (5) and (7). If the curve were of greater radius than 1887 feet, the flange of the outer wheel would recede from the rail until a difference of diameter was produced which should correspond to the radius of the curve, and the wheels would then move onward as before.

It would therefore seem to be a legitimate conclusion, that cars properly furnished with such wheels as we have considered should pass around any curves less than 3° without the resistance due to unequal length of rail, formula (5), and that caused by forcing the car to follow the curve, formula (7). But in order that the wheels of any car may be free to thus adjust themselves to the curvature of the track, it is necessary that the other forces acting in a lateral direction, shall be so nearly in equilibrium that this adjusting power of the wheels, shall be the controlling force in its motion. Let us see if that is the case. The movement of the pair of wheels which we have been considering would be identical with the movement of a frustrum of a cone whose bases were respectively equal in circumference, to the circumference of the wheels. If such a frustrum is placed on a plane surface and put in motion by a force acting horizontally on its centre of gravity and perpendicular to its axis, the path described by that centre of gravity will be a circle. But if the velocity is gradually increased until the centrifugal force becomes greater than the sliding friction between the frustrum and the plane, it will leave its circular path and follow a tangential one. Thus, the practical measure of the centripetal force generated by the cone, is the friction between the cone and the plane upon which it rolls.

Now, if one pair of wheels supporting their share of the weight of a car were placed upon any curved track of less than 3° curvature with the flange nearly, but not quite, in contact with the outer rail, and given a forward motion, they would be influenced by the following forces, acting in a lateral direction. The centripetal force due to the conical shape of the rolling body, the tendency to slide down the inclined plane of the track due to elevation of outer rail, and the radial component of the tractive power, would tend to move it toward the inner rail, while the centrifugal force, and the disposition to press against the outer rail expressed by formula (6), tend to move it toward the outer rail. But if the track is inclined properly with reference to the existing conditions, the tendency to slide down the inclined plane of the track and the centrifugal force will entirely neutralize each other, and the radial component of the tractive power is insignificant in this connection. The pressure towards the outer rail is, as we have already determined, formula (6) $= Wf$, $\times$ by the sine of an angle which can never, in practice, exceed four or five degrees, while the centripetal force due to conicity of wheels is, as we have seen, equal to the friction between the wheels and the rail $= Wf$ — a power so much in excess of the algebraic sum of all the other laterally acting forces as to be amply sufficient to govern the motion of the train.

But in practice it is generally the case that the inclination of the track is not exactly that needed to balance the centrifugal force. The track may be inclined for a maximum speed, while the actual speed may be almost zero, or, on the other hand, the track may have little or no inclination while trains pass at a maximum speed. In the first case, the weight of the car being unbalanced on the inclined plane of the track, would tend to slide inward, and if the preponderance of gravity was sufficient to overcome both the forces acting outward and the friction between the rails and wheels, the flanges would be held against the inner rail, in which case the conicity of the wheels would increase instead of diminishing the train resistance. But if we take any case of this kind likely to occur in practice and compute the magnitude of the various forces acting inward and outward, we will find that the excess of the inward acting forces, is not sufficient to overcome the sliding friction between the wheels and the rails. In the second case, the centrifugal force would be unbalanced ; but as in the first case, we may prove that the excess of outward acting forces is not sufficient to overcome the centripetal force of the coned wheels in cases likely to occur. Hence, we are forced to

conclude that the coned wheels of a car will adjust themselves to any curve whose radius is not less than the altitude of the cone, of which the wheels in the position of the greatest difference of diameter form a frustrum; and since we have found that the wheels used for illustration would, under such circumstances, pass around a three degree curve without slipping or pressing against the outer rail, it is obvious that resistance due to these causes only begins to exist as the curve becomes sharper than three degrees. There is, however, an element of resistance chargeable to coned wheels, which, although it prevails on straight lines as well as curves, it will be proper to consider here; for the modern coned wheel owes its existence to the prevailing belief that the advantages of its use on curved tracks more than compensate for the disadvantage of its use on straight track. This belief, as we shall shall now show, is without proper foundation. If the pair of wheels we have been considering are placed symetrically on a straight track and motion imparted to them, they will roll forward in a straight line. The motion of the system will be similar to that of a cylinder on a plane. But each wheel is itself a truncated cone, and if unrestrained would follow the law of motion characteristic of that body. If compelled to follow a straight line, there will necessarily result a certain amount of resistance.

We have, in a previous section, assumed that the line of contact between the wheel and the rail is one inch in length, and the assumption is probably very nearly correct. The diameter of the wheel at the inner end of this line is greater than at its outer end by an amount equal to $\frac{3}{35}$ of one inch ($4\frac{3}{8}'' : 1'' :: \frac{3}{8}'' : \frac{3}{35}''$) equal to a difference in circumference of 0.02244 feet. It is plain that that part of the wheel which revolves at the outer extremity of the line of contact, being of less circumference, must fall behind the part at the inner end of the line unless the difference is made up by the slipping of the former over a distance equal to the difference of circumference of the two parts. This difference of circumference we have found above to be 0.02244 feet, and as each intermediate part of the wheel between the extremities of line of contact will slip over a space proportioned inversely to its distance from the outer extremity of the line, the average space slipped over by the wheel will be $\frac{0.02244}{2} = 0.01122$ feet, and the work thus done in one revolution of the wheel will be $= W'f + 0.01122$, and if we divide this by the distance gone over in one revolution, we shall have the actual resistance

offered to the progress of the wheels. Calling this resistance g, and the larger diameter of the wheel 33 inches $= 2.75$ feet, we shall have.

$$g = \frac{0.01122\,W'f}{2.75\pi} \qquad (11.)$$

Substituting in this equation for $W'f$ the values, 2000×0.2 we shall find that it expresses a resistance of more than half a pound per ton of the load on the wheels, and it will be readily seen that this resistance is a constant quantity whether the track be straight or curved. We have found that the coning of this pair of wheels will obviate the resistance expressed by formulæ (5) and (7) on curves of three degrees and less, which amounts (see Table No. 2) to less than 0.3 pounds per ton of load, but it now appears that it will create a constant resistance of nearly double that amount. We are therefore forced to the conclusion that the coning of wheels not only does not diminish the train resistance on curves, but that it materially increases the resistance on straight lines.

It is charged and believed by many that to the use of coned wheels is to be attributed the larger part of the violent oscillations and concussions noticed in the motion of trains. The writer believes this to be true, but the question is clearly one to be settled by observation and experiment, and not by analytical investigation.

We have confined ourselves in discussing this subject to wheels of but one form, but the arguments and conclusions deduced will be found equally applicable to those having a greater or a less amount of coning.

If the conclusions we have reached are correct, they furnish sufficient reasons for discarding altogether the use of coned wheels.

§ 10. In addition to the sources of resistance enumerated above, there are several which may be called accidental elements of curve resistance, but as their magnitude can only be determined by experimental investigation, they will only be mentioned here. Bad alignment of track, irregular bends in the rails, uneven joints and ragged rails are much more productive of resistance on curves than on straight lines. Improper inclination of the plane of the track may also be a prolific cause of resistance. If the inclination be too little, the unbalanced centrifugal force of the car may slide the shoulder or end of the axle against the box and thus give rise to frictional resistance. If the inclination be too great the gravity of the car may produce the same result in an opposite direction.

By the aid of the several formulæ deduced above, Table No. 2 has been prepared giving the computed resistance from those sources of various

curves under given conditions. No allowance has been made for the use of coned wheels. In practice, the appropriate amount for accidental sources of resistance, as determined by experiment, should be added to the totals given in the table. This sum added to the previously determined train resistance on straight lines should give the total train resistance on the various curves.

§ 11. A careful inspection of Table No. 2, and the formulæ from which it is calculated, will lead to the following conclusions:

1st. The resistance is very nearly in inverse ratio to the radius of the curve, or directly as the degree of curvature.

2d. It varies directly as the weight of the car.

3d. It is almost entirely independent of the velocity.

4th. Other things being equal, a short car causes slightly less resistance than a long one; but the difference, especially on light curves, is an unimportant part of the whole.

5th. It will be observed that the only formula into which the width of the track g enters as a factor, is formula (5), which expresses the resistance caused by unequal lengths of the outer and inner rails of the track. This element constitutes about 40 per cent. of the whole amount of resistance shown in the table, or, calling this tabular amount the total curve resistance, and assuming the resistance on straight track as $10\frac{1}{2}$ lbs. per ton, about $11\frac{1}{2}$ per cent. of the whole train resistance on a ten degree curve. As the magnitude of this element varies directly with g, it is evident that we may reduce it by reducing the gauge of the track. Thus, if we reduce the standard gauge to three feet, we shall have for a 10° curve:

$$4.7 : 3 :: 205.62 : 131.25 \text{ lbs.},$$

which, added to the other tabular items, makes a total of 452 instead of 526 lbs.; and counting the straight track resistance as before, we find the new value of this element to be 7.7 per cent. of the whole—that is, by reducing the standard to a three-feet gauge, we reduce the train resistance $11\frac{1}{2} - 7.7 = 3.8$ per cent. This conclusion is of interest, in view of the claim made by zealous advocates of the narrow gauge, that short curves are much less objectionable on roads of narrow than of standard gauge.

§ 12. Of the total curve resistance shown by Table No. 2, it will be observed that the element expressed by formula (7) constitutes about 57 per cent. under the conditions assumed. Examination of the formula

Table No. 2.

Showing curve resistance of passenger trains on various curves. Gauge of track 4′ 8½″. Number of cars 6. Weight of each, and load, 40 000 pounds. Length of each over all 52 feet. Length of wheel base of truck (4 wheeled bogie), 6 feet. Velocity 25 miles per hour. Resistance on straight lines assumed to be 10½ pounds per ton of 2 000 pounds.

Degree of Curve.	Weight of Train. (W.)	Actual pressure on rails = W′ Formula (3).	Resistance due to turning of wheel on rail. Formula (4).	Resistance due unequal length of rails. Formula (5.)	Resistance due to pressure of rail in changing direction of motion. Formula (7).	Resistance due to loss of power at each angle of train. Formula (9).	Total resistance of train due to curve.	Resistance in pounds per ton of 2 000 lbs.
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
1°	240 000	240 006	0.1	20.5	29.3	0.2	50.1	0.4
2°	240 000	240 025	0.2	41.0	58.7	0.6	100.5	0.8
3°	240 000	240 057	0.3	61.5	88.0	1.8	151.6	1.3
4°	240 000	240 102	0.5	82.1	117.3	3.4	203.3	1.7
5°	240 000	240 159	0.6	102.6	146.7	5.5	255.4	2.1
6°	240 000	240 229	0.7	123.2	176.1	8.1	308.1	2.6
7°	240 000	240 312	0.8	143.7	205.5	11.4	361.4	3.0
8°	240 000	240 407	0.9	164.3	235.0	15.4	415.6	3.5
9°	240 000	240 515	1.0	184.9	264.5	20.1	470.5	3 9
10°	240 000	240 635	1.2	205.6	294.0	25.6	526.4	4.4

will show that the resistance varies as the sin. of the angle β; and since this angle is small, and varies directly with the length of the wheel base, we may say that this element of resistance varies directly with the length of the rigid wheel base of the car. In view of this fact, and the great comparative magnitude of this source of curve resistance, the importance of making car trucks as short as possible becomes manifest, and the superiority of the American eight-wheeled car, with bogie trucks, over the four-wheeled car, with a long rigid wheel base, is very apparent.

To the general statement above, that the weight of the car being the same, this cause of resistance varies with the length of the wheel base, is to be noted a modification in the case of six-wheeled bogie trucks, in which the central pair of wheels, while they bear their share of the load. do not generate resistance of this kind. To meet such cases, we may substitute in equation (7) for W, $\frac{2}{3} W$, and the formula will then give correct results.

§ 13. Should careful experiments confirm the results reached in this paper, as to the total resistance caused by curved track, it would be very desirable to find some short equation which will give with reasonable accuracy the required resistance without the solution of the several formulae deduced above. As a close approximation, the following may be found convenient:

$$R = D\frac{(g+l)}{25} + an$$

in which,

R = the total curve resistance,

D = the degree of curvature,

g = gauge of the track,

l = length of rigid wheel base, and

an = factors expressing approximately the curve resistance due to accidental causes—to be determined by experiment.

In conclusion, the writer is not ignorant of the fact that the results herein reached vary widely from the opinions generally held by engineers on the subject. The commonly accepted estimate for curve resistance is, that a curve of 10° about doubles the resistance encountered on straight track, while the result arrived at in this paper (see Table No. 2) is less than half that amount. Even when a liberal allowance for accidental resistance is added to the tabular amount, the disparity is too

great to be reconciled. It is very apparent that if the generally received ideas are correct, either the assumptions and arguments of the writer are erroneous or some important elements of resistance have eluded his perception. He readily admits that the weight of evidence is against him; but it will be admitted on the other hand that the experiments on which the conclusions generally held are based, were unsatisfactory, sometimes contradictory, and always inconclusive. It is not impossible that exhaustive experiments, made under the conditions assumed in computing Table No. 2, may give results not very dissimilar from those there given. Careful experiments in this field are greatly needed, and it is to be hoped that the need will be supplied at an early day. In the mean time, the writer's labor will not be in vain if it shall incite a discussion of the subject, and secure its further investigation by competent hands.